Year 5. Problem Set 121 (2009-2010 school year).

1. Find the quotient and remainder of the polynomial $P(x) = x^6 - 4x^4 + x^3 - 2x^2 + 5$ by the polynomial $x + 3$.

2. Prove that $a^2 \frac{(x-b)(x-c)}{(a-b)(a-c)} + b^2 \frac{(x-c)(x-a)}{(b-c)(b-a)} + c^2 \frac{(x-a)(x-b)}{(c-a)(c-b)} = x^2$

3. Rewrite $A(x)/B(x)$ as a sum of a polynomial and a proper fraction:. $A9x) = x^5 - x^3 + 2x^2 - 4$, $B(x) = x^2 - x + 1$.

4. Suppose $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$ is a polynomial with ***integer*** coefficients. Suppose also that a_0 and a_n are nonzero. Assume that $x = p/q$ is a rational root of this polynomial written in the lowest terms. Prove that p is an integer factor of the constant term a_0 and q is an integer factor of the leading coefficient a_n.

5. Solve the encrypted division problem. Each star stands in place of a digit from 0 to 9.

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