Year 5. Session 119 (2009-2010 school year).
Polynomials.

$f(x) = a_{n+}x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$
A polynomial is a sum of terms. Each term is a product of a constant (coefficient) and a variable in a non-negative integer power.
Two polynomials are equal if their coefficients coincide.

Degree of a polynomial $\deg(f)$ is the largest degree of any term.

Multiplication of polynomials

$P(x) = 4x^3 - 7x^2 + 3x - 11, \quad \deg(P) = 3$
$Q(x) = 3x^5 - 2x^4 + 2, \quad \deg(Q) = 5$

$P(x) * Q(x) =$

$\deg\big(P(x) * Q(x)\big) =$

Division of polynomials

How could we divide polynomials?

Polynomial $f(x)$ is divisible by polynomial $g(x)$ if there exist a polynomial $q(x)$ such that:
$f(x) = g(x)q(x)$

Examples:

Remainders:

$f(x) = g(x)q(x) + r(x)$, где $\deg(r) < \deg(g)$.

$r(x)$ – remainder, $q(x)$ quotient.

Example:

Suppose we'd like to divide $f(x)$ by $g(x)$.
$f(x) = 2x^6 - 3x^4 - 5x^3 + x - 6$
$g(x) = x^4 + 3x^3 + 5$

$2x^6 - 3x^4 - 5x^3 + x - 6 = (x^4 + 3x^3 + 5)q(x) + r(x)$

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Another way:

$$2x^6 - 3x^4 - 5x^3 + x - 6 \mid x^4 + 3x^3 + 5$$

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Bezout Theorem

Suppose $P(x)$ is a polynomial . and we would like to divide it by a polynomial of degree 1 $Q(x) = x - a$.

Then the remainder of the division of $P(x)$ by $Q(x)$ equals to $P(a)$ (the value of the polynomial P(x) when $x = a$).

Proof:`

Def: a number a is a root of the polynomial $P(x)$ if $P(a) = 0$.

Corollary1:

A number a is a root of the polynomial $P(x)$ ($P(a) = 0$) if and only if the polynomial $P(x)$ is divisible by $(x - a)$

Proof:

Corollary2:

Suppose $a_1, a_2, \dots, a_k$ are different numbers. Then the polynomial $P(x)$ is divisible by $(x - a_1)(x - a_2) \dots (x - a_k)$ if and only if when all these numbers are the roots of this polynomial.

Proof:
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Example: prove that $x^4 - 2x^3 - 16x^2 + 2x + 15$ is divisible by $(x - 5)(x + 3)(x - 1)$

Corollary3:

A polynomial of degree n cannot have more that n roots.

Example:

Prove that $\frac{(x-a)(x-c)}{(a-b)(a-c)} + \text{ t } \frac{(x-c)(x-a)}{(b-c)(b-a)} + \text{t } \frac{(x-a)(x-b)}{(c-a)(c-b)} = 1$