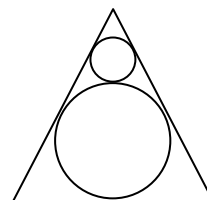## Year 5.   Problem Set 117 (2009-2010 school year).

1. We call a bus filled beyond capacity if it has more than 50 passengers. A motorcade of buses transport a bunch of lazy boys to Pleasure Island where these boys will be transformed into the donkeys. Some of the buses are filled beyond capacity. The nasty Coachmen calculates two numbers. The first one is the percentage of buses that are filled beyond capacity. The second one is the percentage of boys that are traveling in the overfilled buses. Which number is bigger?

2. One hundred and ten scientists from various countries arrived to a conference. They speak in many different languages. It turns out that any three scientists can communicate within a group. Either all three know a same language, or one scientist can translate for other two. Prove that it is possible to split up these scientists into the pairs in such a way that the scientists in each pair can communicate.

3. The triangle ABC is equilateral. Its sides are of unit length. Calculate the radius of the smaller circle.

4. Since 24 = 3+5+7+9, the number 24 can be written as the sum of at least two consecutive odd positive integers.
   (a) Can 2005 be written as the sum of at least two consecutive odd positive integers? If yes, give an example of how it can be done. If no, provide a proof why not.
   (b) Can 2006 be written as the sum of at least two consecutive odd positive integers? If yes, give an example of how it can be done. If no, provide a proof why not.

5. In triangle *ABC*, choose point A_1 on side *BC*, point B_1 on side *CA*, and point C_1 on side *AB* in such a way that the three segments AA_1, BB_1, and CC_1 intersect in one point *P*. Prove that *P* is the centroid of triangle *ABC* if and only if *P* is the centroid of triangle $A_1B_1C_1$.
   Note: A median in a triangle is a segment connecting a vertex of the triangle with the midpoint of the opposite side. The centroid of a triangle is the intersection point of the three medians of the triangle.
   The centroid of a triangle is also known by the names "center of mass" and "medicenter" of the triangle.