Year 5. Problem Set 113 (2009-2010 school year).

1. A paper rectangle of size 6×11 has been cut into a few pieces of two types:  and  Somehow, one of these pieces was lost. Prove one can not assemble a rectangle from the remaining pieces.

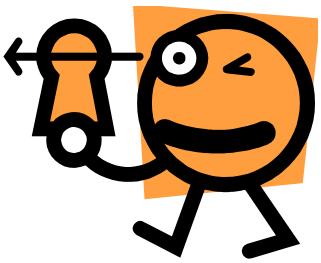
2. A set of open-ended non-intersecting segments was constucted on the line. Prove that this set is not more than countable.

3. Prove that the segment $(0,1)$ has the same cardinality as the segment $[0,1)$.

4. Divide a segment into three smaller segments with length ratios $1:4:7$ using a compass and a non-marked ruler.

5. In a country of NeverEver, there are an infinite number of inhabitants. Any subset of people of this country forms a secret society.

The king of this country would like to organize a system of informants. There are certain principles he would like to follow.



The king wants to have an informant on every secret society. He also requires every inhabitant to be an informer on some society. Lastly, he doesn't want to anybody to report on more then one society.

Do you think the king can succeed?