Year 5. Session 112 (2009-2010 school year). Set Theory.

Reminders:

Def: Two infinite sets *A* and *B* are *equal* if there it is possible to find one-to-one correspondence between the elements of these sets. *Terminology:* Sets *A* and *B* have *same cardinality*, Sets *A* and *B* have *are equivalent*.

Def: Any set that is equivalent to the set of natural numbers \mathbb{N} is called *countable*.

Theorem: The set of all rational numbers is countable. (The set of natural numbers is equivalent to the set of rational numbers).

Theorem: A sum of any number of countable sets is countable.

Non-equal sets.

Lemma: Any infinite set has a countable subset. *Proof:*

Theorem: Countable set is a smallest of all the infinite sets. (A cardinality of a countable set is not greater that the cardinality of any other infinite set). *Proof:*

Theorem: Cardinality of an infinite set does not change if we add a countable set to it. *Proof:*

Theorem: Cardinality of an uncountable set does not change if we subtract a countable set from it.

Proof:

Question: Are all infinite sets countable? Cantor's diagonal argument (1877)

Theorem: The set of all points on the line is uncountable.

Question: which of two segments below has more points?