Year 5. Problem Set 108 (2009-2010 school year).

- 1. Prove that $\log_{10} 11$ is irrational.
- 2. Could it be that
 - a) The sum of two rational numbers is irrational?
 - b) The sum of two irrational numbers is rational?
 - c) An irrational number to an irrational power is rational?
- 3. Set A consists of all integers that are divisible by 4. Set B is composed of all integers divisible by 10. Set C is a set of all integers that are divisible by 75. Describe the numbers that compose the set $A \cap B \cap C$.
- 4. A certain country is populated by magicians, shamans and wizards. It is known that not all magicians are shamans. It is also known that if a wizard is not a shaman, than he is not a magician. Is it true that not all magicians are wizards?
- 5. 10 boxes contain colored pencils. No box is empty. It is known that all the boxes contain different amounts of pencils and that all the pencils in every box are of different color. Prove that it is possible to select a pencil from each box in such a way that all the selected pencils are of different color.
- 6. How many integers between 1 and 1000000 are neither squares, nor cubes nor fourth powers of integers?



