Year 4 Problem Set 99 (2008-2009 school year) Math Battle: Seniors vs. Sophomores

Problems

- 1. In triangle *ABC*, points *D* and *E* divide the side *BC* in three equal parts. Is it possible that two rays, *AD* and *AE*, divide the angle *BAC* into three equal parts as well?
- 2. 9 rooks were placed on a 9×9 board in such manner that no rook could take another. Each rook has been moved into an adjacent square of the board. Prove that now there exists a pair of rooks that can take one other.
- 3. A rectangle is cut into three triangles. Prove that the area of one of these triangles is equal to the sum of the areas of the other two.
- 4. Can you place the numbers 14, 27, 36, 57, 178, 467, 590, 2345 around a circle in such way that any two neighboring numbers would have a common digit? Find all ways to place these numbers which would have this property.
- 5. Prove that out of any 8 natural numbers one can always select two in such way that the difference of squares of these two numbers is divisible by 13.
- 6. The teacher thinks of two consecutive natural numbers. He tells these numbers to two friends, one number each, but he does not tell them which number, the greater or the smaller one, they know. Now our two friends have the following conversation:

First: I do not know what number you have. Second: I do not know your number either. First: I still do not know your number. Second: Same here. First: Then I know what your number is! Second: Then I know as well...

Find what these two numbers could be.