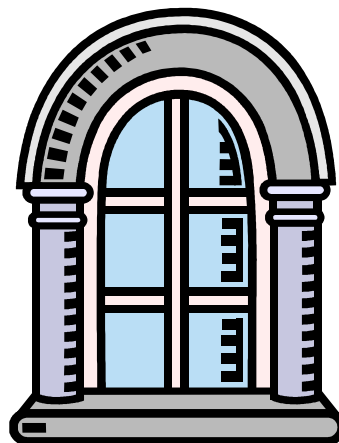


**Year 4 Problem Set 96 (2008-2009 school year)**

1. A plane is colored in three colors. Prove that it is possible to find two points of a same color that are located at distance 1.
2. Find all three-digit numbers  $n$  such that  $n^2 - 2n - 100$  is divisible by 101.
3. For what value of  $x$  does  $1 + 2x + 4x^2 + 8x^3 + 16x^4 + \dots = 10$  ?
4. Is there a set of numbers such that their sum is 1, and the sum of their squares is less than 0.01?

5. You are designing a medieval castle for a king. The king wants all 3 windows in his throne room to have the shape of a rectangle toppled by a semi-sphere with the ends at the upper vertices of the rectangle. Unfortunately, the king wants to place a golden wire inlay around the perimeter of all windows, and the state treasury is very short of gold. If the finance minister is ready to allocate you 4 meters of wire per window, what window dimension of should be in order to let in us much light as possible.



6. A searchlight lights a 90-degrees angle. Prove that for any 4 points on a plane it is possible to place 4 searchlights at these points in such a way that these searchlights would light up all the plane.
7. A convex  $n$ -vertex polygon is divided by its diagonals into triangles. The diagonals are chosen in such a way that they don't intersect except, probably, at the ends. It is also known that every vertex of the polygon is a vertex for an odd number of such triangles. Prove that  $n$  is divisible by 3.