Year 4 Problem Set 94 (2008-2009 school year)

- 1. How many positive 20-digit integers that are written using only 0s and 5s are divisible by 9?
- 2. Prove that $(a + b)(b + c)(c + a) \ge 8abc$ for any $a, b, c \ge 0$.
- 3. Prove that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \ge \frac{1}{\sqrt{ab}} + \frac{1}{\sqrt{bc}} + \frac{1}{\sqrt{ca}}$ for any $a, b, c \ge 0$.
- 4. Prove that $a^4 + b^4 + c^4 \ge abc(a + b + c)$ for any $a, b, c \ge 0$.
- 5. The sum of three positive numbers is six. Prove that the sum of their squares is no less than 12.
- 6. For what value of *x* does $1 + x + x^2 + x^3 + x^4 + \dots = 4$?
- 7. A *crocodile-N* is a new chess piece. It moves *N* squares horizontally or vertically, and then 1 square in the perpendicular direction (a knight is a crocodile-2). For which *N* a *crocodile-N* can get from any square of an infinite chess board to any other square?



