Year 4 Problem Set 104 (2008-2009 school year)

- 1. A rectangle with integer sides is tessellated by the following figures: Prove that the same rectangle can be tessellated by the following figures:
- 2. An *irrational* number is a number that can't be represented as a fraction a/b, where a and b are integer. Using this definition, prove that $\sqrt{2}$ is an irrational number.
- 3. Prove that for any non-convex polygon there exists a convex polygon with a smaller perimeter and larger area.
- 4. There are a few very tranquil flies sitting on the table. You can cover any three of them with an upside-down cup. Prove that you can cover all flies in one go using the same cup.



5. A city of Paris is inhabited by Catholics and Protestants. Some of those who are of different religions are, in fact, friends. Each Protestant that has a Catholic friend is trying to convert his friend into his own religion. The same is true for Protestants. Every day either some Catholic who has more

Protestant friends that Catholic friends converts, or a Protestant who has a majority of Catholic friends converts. A king really doesn't like this situation. But the cardinal Richelieu is convincing him that that these conversions will stop in the near future. How does cardinal know this?

