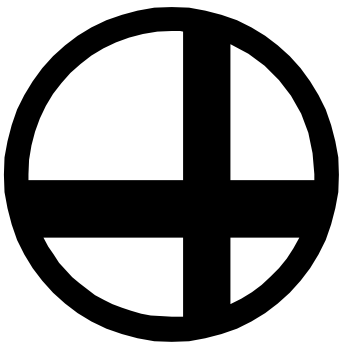
Year 4 Problem Set 103 (2008-2009 school year)

1. A boss has asked his assistant Richard Mayhew to buy a few things for the Christmas office party. He wanted one cake, 3 bottles of champagne and 20 crystal wine glasses. Instead, for exactly the same money, Richard Mayhew bought 1 crystal wine glass, 3 cakes and 20 bottles of champagne. It is well-known that a cake is less expensive than a bottle of champagne. What costs less: bottle of champagne or crystal wine glass?

2. Do you think that an intersection of an infinite number of open sets has to be an open set? (Hint: compare with the problem about the intersection of 1000 open sets).

3. Prove that a set and its complement have a common set of boundary points: every boundary point of a set is a boundary point of its complement, and backward.

4. 100 chords of a single circle all intersect one with another. Prove that is possible to draw one more chord in such a way that it would intersect each of 100 chords. (An intersection an endpoint is counted as an intersection).



5. A table 10×10 contains 100 integer numbers. It is known that the sum of numbers located on the intersection of any 5 columns and rows is even. Prove that all the numbers in the table are even.