Year 4 Class 101 (2008-2009 school year)

Convexity. Part 2.

Def. The figures that contain their own boundary are called ***closed.***
Def. The figures that do not contain their own boundary are called ***open.***

Problem: Provide some examples of bounded and unbounded open and closed sets. Do you think that an empty set is an open set? Closed set?

Problem: Prove that the intersection of two open sets is an open set.

Def. A figure (a set of points) together with all its boundary points is called a ***closure*** of this figure.

Do you think that a closure of a figure is always an open set? Closed set? Not possible to tell?

Some examples of figures and their closures:

Def. Set of all points on the plane that don't belong to a figure are called a ***complement*** of this figure.

$$compelment\ of\ A = \{x: x \notin A\}$$

Can you think about a figure with unbounded complement? Bounded complement? Convex complement? Closed complement? Open Complement?

Def: A line is called a ***supporting line*** to figure A if it intersects the figure, but the entire figure is located to one side of the line.

It is kids of obvious that any convex figure has no more than two parallel supporting lines. Then the figure is kind of sandwiched between these two lines. Could you think of an example of a convex figure that has just one supporting line? No supporting lines?

Theorem. Any bounded convex figure A with a nonempty interior has exactly two supporting lines that are parallel to a given line.

Proof: