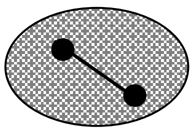
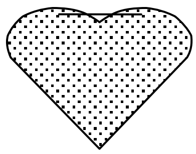
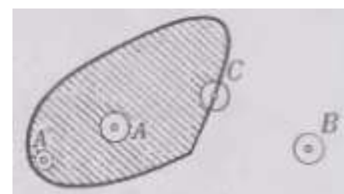
Year 4 Class 100 (2008-2009 school year)

Convexity

Def. A figure A in Euclidean space is ***convex*** if it contains all line segments connecting any pair of its points. If the figure does not contain all such line segments, it is called concave.

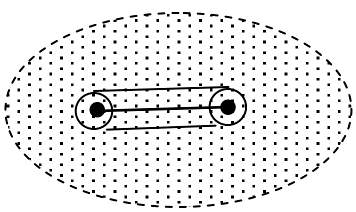



Def. A point x in is an ***inner point*** of a figure A if there exist a small disk with the center at this point such that this disk is fully contained in A.
Def. A point x in is an ***outer point*** of a figure A if there exist a small disk with the center at this point such that this whole disk is located outside of A.
Def. A point x in is a ***boundary point*** of a figure A if it is neither an inner point nor an outer point.



Problem: Prove that a point x in is a boundary point of A if and only if every small disk centered at x contains at least one point in A and at least one point not in A.

Problem: Two inner points of a convex figure are connected by a segment. Prove that all points on this segment are inner points of the figure as well.
Solution:



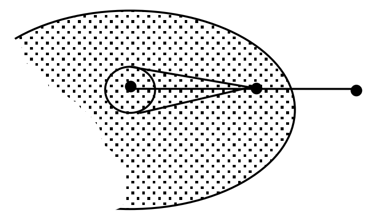
Def: The figures that contain their own boundary are called **closed**. The figures that don't contain their boundaries are called **open**.

From now on, unless stated otherwise, we will only consider figures that contain their own boundary and have inner points (are not lines).

Theorem. If we connect an inner and an outer points of a convex figure *A*, then the segment connecting them actually consists of three parts: a smaller segment that is inside the figure, a smaller segment that is outside the figure, and a single boundary point in between.

> To prove this fact, we'll use *"The Completeness Axiom":* suppose that B and C are two sets of numbers such that any number from the set B is less than any number from the set C. Then there exist a number that separates these two sets: it is not greater than any number from the bigger set, and not less than any number from the smaller set.

Proof: (to be done in class together with students)



For any point on the segment that belong to A, all the points between this point and the inner endpoint belong to A as well. For any point on the segment that don't belong to A, all the points between this point and the inner endpoint belong to A as well.

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