Year 4 Problem Set 100 (2008-2009 school year)

1. Prove that if A is an inner point of a convex figure, and B is a boundary point of this figure, then all the points of a segment connecting A and B are inner points of the figure. Assume that the figure is closed, so the point B belongs to the figure. For an extra point, prove the same fact for any convex figure, not necessary closed.

2. Prove that a segment connecting two *boundary* points of a convex figure is either located completely inside the figure, or completely on the boundary. Assume that the figure is closed, so the boundary points belong to the figure. For an extra point, prove the same fact for any convex figure, not necessary closed.

3. Prove that an intersection of two convex sets is a convex set as well.

4. In a certain triangle ABC, the median, height and bisector that originate at the angle A divide this angle into 4 equal parts. Find the angles of this triangle.

5. A square of size 1000×1000 is divided into rectangles by a few straight lines. The straight lines are all drawn along the grid lines. The resulting rectangles are all painted black or white using a standard chess coloring scheme. As a result, each of the 1.000.000 small squares is painted either black or white. Prove that the number of black squares is even.

6. There are 175 passengers and two conductors on the trains. A passenger will buy a ticket at the moment when he is reminded about the need to buy it for third time. At the start of the trip, none of the passengers have tickets. Conductors take turns reminding passengers one by one about the need to have tickets. What should be the strategy of the first conductor if he wants to sell as many tickets as possible?